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Technical Note TN 23

A note on the effect of a return surge

by

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The object of this note is to show the effect of a "return surge" on the elevation of the North Sea by means of a mathematical model. The geometry of the model is determined by the rectangle $0 < x < a$, $0 < y < b$, where $x=0$, $x=a$, $y=0$ represent coasts and $y=b$ the open end at the ocean. The depth h is given by the exponential function

$$(1) \qquad h = h_o \, e^{\beta y} \, .$$

The numerical values are $a=400$ km, $b=800$ km, $h_o=33$ m, $h(b)=158$ m.

The hydrodynamical equations are

$$(2) \qquad \begin{cases} \left(\frac{\partial}{\partial t} + \lambda\right)u - \Omega v + gh\frac{\partial \zeta}{\partial x} = U \\ \left(\frac{\partial}{\partial t} + \lambda\right)v + \Omega u + gh\frac{\partial \zeta}{\partial y} = V \\ \frac{\partial \zeta}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \end{cases}$$

with the boundary conditions

$$(3) \qquad \begin{cases} u=0 & \text{for} \quad x=0, \quad x=a \\ v=0 & \text{for} \quad y=0 \\ \zeta=0 & \text{for} \quad y=b \end{cases} ,$$

and with the initial condition

$$(4) \qquad u=v=\zeta=0 \qquad \text{for } t=0.$$

The numerical values of the coefficients of friction and rotation are $\lambda=0.09 \ \text{hr}^{-1}$, $\Omega=0.44 \ \text{hr}^{-1}$.

A numerical calculation has been carried out by means of a difference scheme for the particular windfield

$$(5) \qquad \begin{cases} U=0 \\ V = \begin{cases} -\sin\frac{\pi ct}{10\,a} & \text{for} \quad 0 \leq t \leq \frac{10\,a}{c} \\ 0 & \text{for} \quad t > \frac{10\,a}{c} \end{cases} \end{cases}$$

where c denotes the mean velocity of free waves, numerically $c=91$ km/hr. This windfield represents a uniform "Northern" wind varying sinusoidally with a duration of 44 hr (see fig.1).

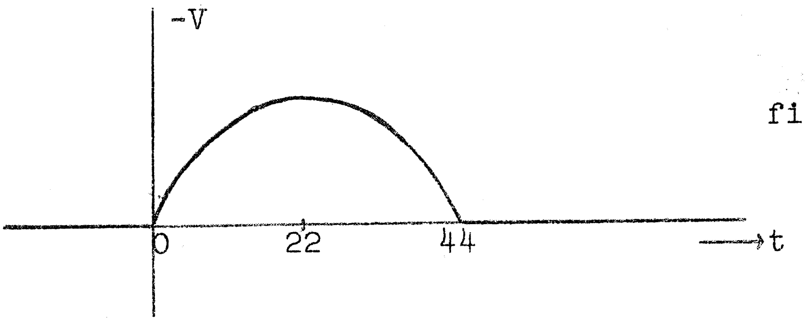

figure 1

Computations have been carried out for a grid with a basic mesh of $\delta=a/24$, i.e. with squares of 16×16 km. The elevations were calculated at all points $x=(2m+1)\delta$, $y=(2n+1)\delta$ with $m=0,1,\ldots,11$, $n=0,1,\ldots,23$.
The following table gives the elevations at the points indicated by small circles in figure 2 at intervals of 2τ or 3τ where $\tau=\frac{a}{\pi c}=1.4$ hour . The elevations are given in centimeters.

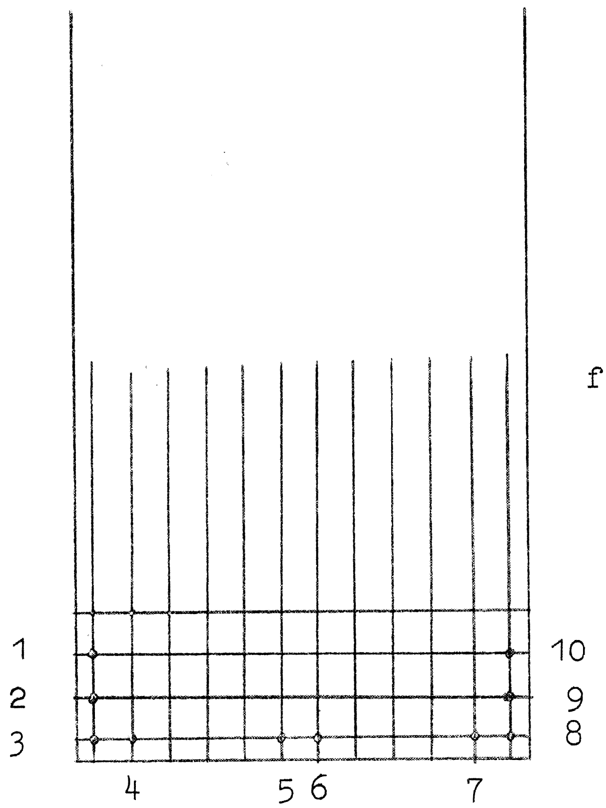

figure 2

Situation of points nos 1,...,10.

point no / time	1	2	3	4	5	6	7	8	9	10
2τ	14	19	26	24	22	22	20	19	10	3
4τ	82	92	107	102	79	75	63	61	42	25
6τ	182	200	224	215	172	160	125	121	93	68
9τ	337	367	404	394	353	342	305	301	256	214
12τ	469	510	556	547	509	498	464	463	410	362
15τ	532	577	627	622	605	601	586	587	531	478
18τ	544	592	642	639	634	632	628	631	578	529
21τ	499	541	585	586	597	600	611	615	570	528
24τ	418	454	489	493	514	519	537	541	508	477
27τ	298	322	345	352	385	393	421	425	406	389
30τ	156	168	177	186	225	235	267	271	269	266
33τ	4	6	5	15	59	71	107	109	117	124
36τ	-50	-53	-57	-52	-38	-33	-15	-14	- 6	1
38τ	-47	-49	-53	-51	-52	-53	-57	-60	-53	-47
40τ	-31	-34	-36	-35	-36	-37	-44	-47	-44	-42
42τ	- 7	- 9	-11	-11	-19	-21	-28	-31	-28	-27
44τ	8	8	8	8	2	0	-10	-12	-11	-11
46τ	9	8	8	8	11	11	10	9	9	8
48τ	12	12	12	12	11	11	13	12	13	14
50τ	7	8	9	9	11	12	11	11	11	12

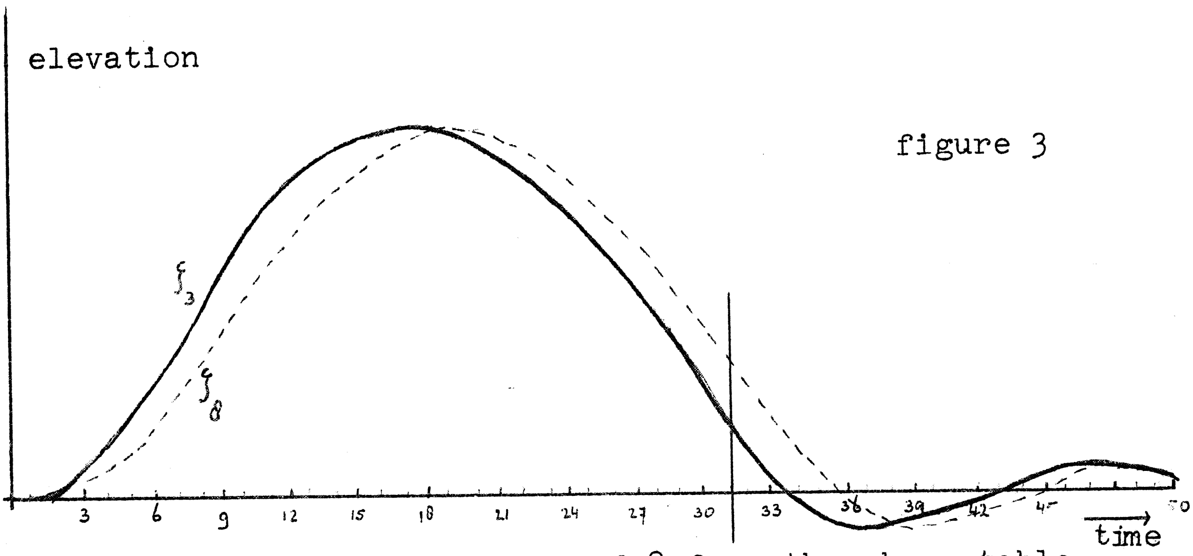

figure 3

Elevation at the points no 3 and 8 from the above table

Similar calculations have been carried out for the windfield

$$(6) \qquad \begin{cases} U = 0 \\ V = \begin{cases} -1 & \text{for} \quad t < 0 \\ 0 & \text{for} \quad t > 0 \end{cases} \end{cases} .$$

This windfield represents a uniform and constant "Northern" wind which stops suddenly at t=0.
At that time the sea is in its equilibrium position

$$(7) \qquad u=v=0 \quad , \quad gh_o\zeta = \frac{e^{-\beta y} - e^{-\beta b}}{\beta} .$$

The following table shows that the subsequent motion has the appearance of a strongly damped oscillation.

point no / time	1	2	3	4	5	6	7	8	9	10
0	510	555	603	603	603	603	603	603	556	511
3τ	167	184	192	210	312	339	383	387	391	396
6τ	-44	-47	-52	-45	16	45	142	150	165	178
9τ	-112	-112	-113	-107	-84	-82	-105	-120	-108	-94
12τ	-63	-80	-90	-89	-108	-113	-99	-95	-83	-72
15τ	- 8	- 5	- 1	5	9	1	-26	-30	-40	-45
18τ	30	27	29	23	4	1	14	16	17	22
21τ	9	11	12	14	23	22	21	21	21	20
24τ	11	15	17	17	15	18	19	19	22	24
27τ	-14	-10	- 5	- 4	1	- 1	-13	-14	-12	- 8
30τ	- 3	- 1	1	1	- 1	0	9	10	8	7

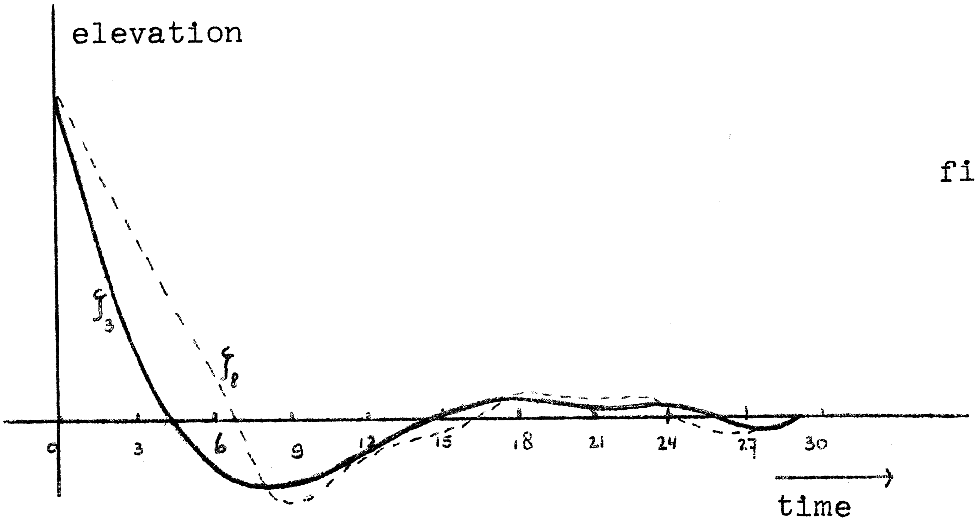

figure 4

Elevation at the points 3 and 8 from the above table.

These two cases indicate that any disturbance of the elevation is damped out very rapidly.

Considering figure 3 first, we see that after the wind has stopped the elevation has a negative extremum which is about 10% of the previous maximum.

If all signs are inverted we obtain the following conclusion.

A uniform sinusoidal "Southern" wind of the type (5) brings about a low water followed by a high water (return surge) the intensity of which is of the order of 10% of the previous low tide.

Considering next figure 4 we arrive at a similar conclusion:

If a uniform and constant "Southern" wind blows sufficiently long as to bring the sea in its equilibrium position and then suddenly stops, then after an interval of 12 to 15 hours a return surge develops, the intensity of which is of the order of 20% of the previous minimum.

Finally an illustrative numerical example will be given. For a uniform "Southern" wind of the type (5) with a maximum of 30 m/sec a return surge of about 0.33 m. is obtained after an initial low water of about -3.40 m. For a uniform and constant "Southern" wind of 30 m/sec a return surge of about

0.60 m. is obtained after an initial low water of about -3.20 m.

References

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